# 量子化学理论及其在催化反应中的应用



催化基础国家重点实验室 2007-7

#### Heterogeneous catalysis



Free energy

Worldwide turnover ~14 Trillion US\$ (2000)

> 90% of all chemicals pass a catalyst once during production



### Catalysis: a playground on many levels







washcoat support



ceramic monolith cells  ${\sim}500~\mu\text{m}$ 



ceramic monolith



washcoat macropores ~1-10 μm





single crystal surface

# Supported Catalyst

#### Structural Parameters



Kinetic Effects

### **Multiscale Process**



of surface properties and functions

K. Reuter, C. Stampfl and M. Scheffler,

in: Handbook of Materials Modeling, Part A. Methods,

(Ed.) Sidney Yip, Springer (Berlin, 2005).

http://www.fhi-berlin.mpg.de/th/paper.html

### Sabatier principle



J.R. Jennings, *Ammonia Synthesis*, Plenum (1991)





M. Todorova *et al.*, Phys. Rev. Lett. 89, 096103 (2002)

### Electronic structure theory calculations of surfaces

Tight-binding **Density-functional theory** Quantum chemical methods (> HF) (Quantum Monte Carlo)

- Forces (relaxation, vibrations, MD...)

- Total energy

- Electronic structure (...)

VS.











#### From micro to meso



(real world...?)

# Outline

### I. Introduction

- ① Density functional theory
- 2 Ab initio atomistic thermodynamics
- ③ Transition state theory

### II. Examples

- A. Interaction between molecules and metal surface
- B. Manipulation of electron and reactivity: QSE
- C. Oxidation of Pt(332) (DFT + XPS)
- D. Reduction of nano-structural surface (DFT+STM)
- E. Bridging pressures and materials gap
- F. Ammonia Synthesis
- G. Selectivity
- H. Screening and rational catalysis design

# Fast Guide to Density Functional Theory

# Walter Kohn, Nobel Prize 1998







"Self-consistent Equations including Exchange and Correlation Effects" W. Kohn and L. J. Sham, Phys. Rev. 140, A1133 (1965)

Literal quote from Kohn and Sham's paper:"... We do not expect an accurate description of chemical binding."

# 研究方法:密度泛函理论(DFT)



#### General 'condensed-matter' Hamiltonian

 $(\hat{T}^e + \hat{T}^{ion} + \hat{V}^{e-e} + \hat{V}^{e-ion} + \hat{V}^{ion-ion})\Psi = E\Psi$ wavefunction  $\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_N; \mathbf{R}_{\mathbf{I}}, \cdots, \mathbf{R}_{\mathbf{M}})$ electronic coordinates  $\mathbf{r}_{\mathbf{k}}, k = 1, \dots N$ ionic coordinates  $\mathbf{R}_{\mathbf{I}}$ ,  $I = 1, \cdots M$  $\hat{T}^e = \sum_{k=1}^{N} \frac{\mathbf{p}_k^2}{2m} \qquad \qquad \hat{T}^{ion} = \sum_{I=1}^{N} \frac{\mathbf{P}_I^2}{2M_I}$  $\hat{V}^{e-e} = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \sum_{k \neq k'}^{N,N} \frac{e^2}{|\mathbf{r_k} - \mathbf{r_{k'}}|}$  $\hat{V}^{ion-ion} = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \sum_{I \neq I'}^{M,M} \frac{Z_I Z_{I'}}{|\mathbf{R}_{\mathbf{I}} - \mathbf{R}_{\mathbf{I}'}|}$  $\hat{V}^{e-ion}(\mathbf{r_{k}}, \mathbf{R_{I}}) = \sum_{i=1}^{N} \sum_{j=1}^{M} \mathbf{v_{I}^{ion}}(|\mathbf{R_{I}} - \mathbf{r_{k}}|)$ 

## **Born-Oppenheimer approximation**

convenient, frequently made approximation (but not compulsary) separation of variables  $\rightarrow$  parametric dependence on set of coord.  $\{R_I\}$ 

$$\Psi(\mathbf{r_1},\cdots\mathbf{r_N};\mathbf{R_1},\cdots\mathbf{R_M}) = \sum_{\nu} \Lambda_{\nu}(\{\mathbf{R_I}\}) \Phi_{\nu,\{\mathbf{R_I}\}}(\mathbf{r_k})$$

electronic Schrödinger equation

$$H^{e}_{\{\mathbf{R}_{\mathbf{I}}\}}\Phi_{\nu,\{\mathbf{R}_{\mathbf{I}}\}}(\mathbf{r}_{\mathbf{k}}) = E^{e}_{\nu,\{\mathbf{R}_{\mathbf{I}}\}}\Phi_{\nu,\{\mathbf{R}_{\mathbf{I}}\}}(\mathbf{r}_{\mathbf{k}})$$
$$H^{e} = T^{e} + V^{e-e} + V^{e-ion}$$

frequently made approximations:

- neglect of non-adiabatic couplings (terms of order  $m/M_I$ )
- only one  $\Lambda_{\nu} \neq 0$  (in a solid, this means neglect of electron-phonon couplings)

 $\rightarrow$  electronic and nuclear degrees of freedom decouple !

# **Limitations of Born-Oppenheimer**

- doesn't account for correlated dynamics of ionic and electronic coordinates
   *Example:* suprafluid He3, polaron-induced superconductivity
- breakdown of the restriction to a single ground-state Born-Oppenheimer surface *Example:* chemoluminescence
- breakdown of the adiabatic approximation
   *Example:* excitation of surface plasmons during scattering of an ion from a
   metal surface → time-dependent density functional theory

#### electronic many-particle Hamiltonian

$$\begin{bmatrix} \sum_{k=1}^{N} \frac{\nabla_k^2}{2m} + v^{(0)}(\mathbf{r_k}) + \sum_{k \neq k'}^{N,N} \frac{1}{2} W(\mathbf{r_k}, \mathbf{r'_k}) \end{bmatrix} \Phi(\mathbf{r_1}, \cdots \mathbf{r_N}) = E\Phi(\mathbf{r_1}, \cdots \mathbf{r_N})$$
$$W(\mathbf{r}, \mathbf{r'}) = \frac{e^2}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r'}|}$$
$$v^{(0)}(\mathbf{r}) = \sum_{I=1}^{M} v_I^{ion}(|\mathbf{R_I} - \mathbf{r}|)$$

still many (for a typical solid: 10<sup>23</sup>) degrees of freedom

The many-particle problem can be solved only for small systems (atoms, molecules and clusters).

 $\rightarrow$  wavefunction-based methods

- Configuration-interaction, Coupled Cluster method
- Quantum Monte Carlo method

# **Density Functional Theory**

#### Hohenberg-Kohn theorem:

For any given external potential  $v^{(0)}$ , the wavefunctions can be considered as functionals in the space of ground state densities, *n*:

$$E_{v^{(0)}}[n] = \langle \Phi[n] | \hat{T}^e + \frac{1}{2} \hat{W} + \hat{v}^{(0)} | \Phi[n] \rangle$$

The energy functional is stationary at the ground state energy, and the true ground state density  $n_0$  coincides with n at the stationary point. Therefore a universal functional  $\mathcal{F}$  exists with the property

$$E_{v^{(0)}}[n] = \mathcal{F}[n] + \int d\mathbf{r} \ v^{(0)}(\mathbf{r})n(\mathbf{r})$$

Proof:

 $\begin{array}{ll} {}^{\prime}\Phi[n] \mapsto n' \colon & \text{trivial} \\ {}^{\prime}n \mapsto \Phi[n]' \colon & \text{Note that } v_{ext} \to n \stackrel{\text{Raleigh}-\text{Ritz}}{\mapsto} \Phi[n] \\ & \text{Also: } v_{ext} \mapsto \Phi[n]. \text{ Thus, } E_{v^{(0)}}[n] \text{ is uniquely defined.} \end{array}$ 

# Kohn-Sham theorem

idea: decompose  $\mathcal{F}[n]$  into its major contributions  $\mathcal{F}[n] = T_0[n] + \frac{1}{2} \int \int d\mathbf{r} \, d\mathbf{r}' \, n(\mathbf{r}) W(\mathbf{r}, \mathbf{r}') n(\mathbf{r}') + E_{XC}[n]$   $T_0$ : kinetic energy of a system of non-interacting particles  $E_{XC}[n]$  exchange-correlation energy defined as 'the rest'  $\rightarrow$  approximations  $V_{XC}[n](r) := \frac{\delta E_{XC}[n]}{\delta n(r)}$ : exchange-correlation potential

popular approximations for  $E_{XC}[n]$ :

•

- local-density approximation (LDA)
- generalized gradient approximations (GGAs)
- exact exchange formalism (EXX)

# Kohn-Sham Hamiltonian

To find the stationary point, we do variations at fixed  $N = \int d\mathbf{r} n(\mathbf{r})$ , which leads to

$$\frac{\delta E_{v^{(0)}}}{\delta n(\mathbf{r})} = \mu$$
 (Lagrange parameter)

If we write the density as a sum over single-particle functions,

$$n(\mathbf{r}) = \sum_{\mathbf{j}=1}^{\mathbf{N}} \sum_{\mathbf{k} \in \mathbf{BZ}} |\mathbf{'}_{\mathbf{j},\mathbf{k}}(\mathbf{r})|^{2},$$

the variational principle  $\delta E_{v^{(0)}}[\varphi^*]/\delta \varphi^*(\mathbf{r}) = \mathbf{0}$  leads to the Kohn-Sham equations

$$\left(-\frac{\nabla^2}{2m} + V_{\text{eff}}[n](\mathbf{r})\right)\varphi_{j,\mathbf{k}}(\mathbf{r}) = \epsilon_{j,\mathbf{k}}\varphi_{j,\mathbf{k}}(\mathbf{r})$$

with the effective potential

$$V_{\text{eff}}[n](\mathbf{r}) = v_{R_I}^{(0)}(\mathbf{r}) + \int d\mathbf{r}' \frac{e^2 n(\mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|} + \frac{\delta E_{XC}[n]}{\delta n}(\mathbf{r}).$$

# How can we specify $E_{XC}$ ?

rs

At this point, we need to make approximations to get further. *Example:* local density approximation (LDA)

$$\begin{split} E_{\rm XC}[n(\mathbf{r})] &= \int d\mathbf{r} \; e_{\rm XC}[n(\mathbf{r})] \; n(\mathbf{r}) \\ &\approx \int d\mathbf{r} \; [e_X^{hom}(n(\mathbf{r})) + e_C^{hom}(n(\mathbf{r}))] \; n(\mathbf{r}) \\ e_X^{hom}(n) &= -(81/64\pi)^{1/3} n^{1/3}(\mathbf{r}) \\ e_C^{hom}(n) &= \begin{cases} -0.1423(1+1.0529\sqrt{r_s}+0.3334r_s)^{-1} & \text{if} \; r_s \ge 1, \\ -0.0480+0.0311 \ln r_s - 0.0116r_s + 0.002 \; r_s \; \ln r_s \\ & \text{if} \; r_s < 1. \end{cases} \\ r_s := (4\pi n(\mathbf{r})/3)^{-1/3} \quad \text{Wigner-Seitz radius} \\ \text{[see, e.g. J. Perdew & A. Zunger, Phys. Rev. B 23 5048 (1981)]} \end{split}$$

# The total energy (for static ions)

two equivalent definitions:

$$E_{tot}[n] = T_0[n] + \int d\mathbf{r} \, v_{\mathbf{R}_i}^{(0)}(\mathbf{r}) n(\mathbf{r}) + \frac{1}{4\pi\epsilon_0} \int d\mathbf{r} \int d\mathbf{r}' \, \frac{e^2 n(\mathbf{r}) n(\mathbf{r}')}{2|\mathbf{r} - \mathbf{r}'|} + E_{XC}[n] + V_{\mathbf{R}_I}^{ion-ion}$$

$$E_{tot}[n] = \sum_{j=1}^N \sum_{\mathbf{k} \in BZ} \varepsilon_{j,\mathbf{k}} + \Delta E^{e-e}[n] + \Delta E_{XC}[n] + V_{\mathbf{R}_I}^{ion-ion}$$

$$\Delta E^{e-e}[n] = -\frac{1}{4\pi\epsilon_0} \int d\mathbf{r} \int d\mathbf{r}' \, \frac{e^2 n(\mathbf{r}) n(\mathbf{r}')}{2|\mathbf{r} - \mathbf{r}'|} = -E^{e-e}[n]$$

$$\Delta E_{XC}[n] = E_{XC}[n] - \int d\mathbf{r} \, V_{XC}[n](\mathbf{r}) \mathbf{n}(\mathbf{r})$$

 $E_{tot}$  is stationary with resp. to variations of *n* around  $n_0$ , but the individual terms are not !

# Concepts when solving Schrödingers-equation in solids



#### Basic steps in an electronic structure calculation

- guess a starting charge density (e.g. superposition of atomic densities)
- 2. set up the Hamiltonian for this charge density (usually done in a small, preliminary basis set)
- 3. diagonalize this approximative Hamiltonian
- 4. use the eigenvalues and wavefunctions to set up a new charge density
- 5. try to improve the wavefunctions using the variational principle for  $E_{tot}$ , thereby simultaneously approaching self-consistency

$$n^{(i-1)} \to V_{eff}^{(i-1)} \to \varepsilon_{j,\mathbf{k}}^{(i-1)}, \varphi_{j,\mathbf{k}}^{(i-1)} \to n^{(i)} \to V_{eff}^{(i)} \to \varepsilon_{j,\mathbf{k}'}^{(i)}, \varphi_{j,\mathbf{k}'}^{(i)} \cdots$$

### Surface models: slabs vs. clusters



VS.



#### <u>Cluster geometries:</u>

- + very cheap for small clusters (local basis sets)
- + ideal for local aspects (defects etc.)
- slow convergence with cluster size (embedding etc.)

#### Supercell geometries:

- + proper surface electronic structure (band structure)
- + good convergence with slab thickness ("semi-infinite")
- + suitable for plane wave basis sets
- artificial lateral periodicity: "ordered arrays"
- inherently expensive (large systems)





# **Basis Sets for Solids**

- plane waves (pseudo potentials)
- space partitioning (augmentation) methods
  - LMTO (linear muffin tin orbitals)
    - ASA approx., linearized numerical radial function
    - + Hankel- and Bessel function expansions
  - ASW (augmented spherical wave)
    - similar to LMTO
  - FP-LMTO (full-potential LMTO)
    - similar to LAPW, space partitioned with non-overlapping spheres
  - KKR (Kohn, Koringa, Rostocker method)
    - solution of multiple scattering problem, Greens function formalism
    - equivalent to APW
  - (L)APW (linearized augmented plane waves)
- LCAO methods
  - Gaussians, Slater, or numerical orbitals, often with PP option)

# pseudopotential plane wave methods

• plane waves form a "complete" basis set, however, they "never" converge due to the rapid oscillations of the atomic wave functions  $\chi$  close to the nuclei

• let's get rid of all core electrons and these oscillations by replacing the strong ion-electron potential by a much weaker (and physically dubious) *pseudopotential* 



# How to compute observables

A) energy differences

between different structures, formation energy of defects, heat of adsorption, ...

**B**) derivatives of the thermodynamic potentials  $E_{tot}$ , F or GFor simplicity's sake, we consider a system with constant volume at T = 0:  $F(T = 0, V) = E_{tot}(V)$ 

• pressure 
$$p = -\frac{\partial E_{tot}(V)}{\partial V}$$

• bulk modulus 
$$B = V \frac{\partial^2 E_{tot}(V)}{\partial V^2}$$

• forces  $\vec{F}_I$  on atom *I* (in electronic ground state)

$$\vec{F}_{I} = -\frac{\partial E_{tot}}{\partial \vec{R}_{I}} = -\sum_{j}^{N} \langle \varphi_{j,\mathbf{k}} | \partial H / \partial \vec{R}_{I} | \varphi_{j,\mathbf{k}} \rangle$$

due to the Hellmann-Feynman theorem

## How to compute observables

- **C**) second derivatives *Examples:* 
  - 1. force constant matrix
    - $\frac{\partial E_{tot}}{\partial \vec{R}_i \partial \vec{R}_j}$

ightarrow calculation of phonon spectrum, vibrational entropy,  $\dots$ 

2. particle number fluctuations

 $\rightarrow$  chemical softness and hardness

$$s(\mathbf{r}) = \left(\frac{\partial n(\mathbf{r})}{\partial \mu}\right)\Big|_{v,T} = \int \left(\frac{\delta^2 E_{tot}}{\delta n(\mathbf{r})\delta n(\mathbf{r}')}\right)^{-1} d\mathbf{r}'$$
$$h(\mathbf{r}) = \frac{1}{2} \left(\frac{\partial \mu}{\partial n(\mathbf{r})}\right)\Big|_{v,T} = \int \frac{\delta^2 E_{tot}}{\delta n(\mathbf{r})\delta n(\mathbf{r}')} \frac{n(\mathbf{r}')}{2N} d\mathbf{r}'$$

Note: When calculating second derivatives, the response of the density to the perturbation must be taken into account.  $\rightarrow$  Density Functional Perturbation Theory

#### What materials properties are accessible to calculation?

- structural properties *Examples:* structural phase transitions, surface reconstructions **yes**
- elastic properties *Examples:* bulk modulus,  $C_{11}$ ,  $C_{12}$ ,  $C_{44}$ , ...
- chemical properties
   *Examples:* thermochemical stability of compounds, reactivity of surfaces
   yes
- transport properties *Examples:* conductance of nanowires, magneto-resistence

developing field

yes

optical/spectroscopic properties
 *Examples:* photoemission spectra, cross sections for light absorption
 topic beyond Kohn-Sham theory → time-dependent DFT

many other applications

Bridging scales: Ab initio atomistic thermodynamics

### **General idea**

#### Motivation:

- extend length scale
- consider finite temperature effects
- Approach:
  - separate system into sub-systems (exploit idea of reservoirs!)
  - calculate properties of sub-systems separately (cheaper...)
  - connect by implying equilibrium between sub-systems

#### Drawback:

- no temporal information
  - ("system properties after infinite time")
- equilibrium assumption



I. Connecting thermodynamics, statistical mechanics and density-functional theory

> *Statistical Mechanics,* D.A. McQuarrie, Harper Collins Publ. (1976)

Introduction to Modern Statistical Mechanics, D. Chandler, Oxford Univ. Press (1987)

M. Scheffler in *Physics of Solid Surfaces 1987*, J. Koukal (Ed.), Elsevier (1988)

### **Thermodynamics in a nutshell**

Internal energy(U) $E^{tot}(S,V)$ Enthalpy $H(S,p) = E^{tot} + pV$ (Helmholtz) free energy $F(T,V) = E^{tot} - TS$ Gibbs free energy $G(T,p) = E^{tot} - TS + pV$ 

**Potential functions** 

- Equilibrium state of system minimizes corresponding potential function

- In its set of variables the total derivative of each potential function is simple (derive from 1<sup>st</sup> law of ThD:  $dE^{tot} = dQ + dW$ , dW = -pdV, dQ = TdS)

- Chemical potential  $\mu = (\partial G / \partial n)_{T,p}$  is the cost to remove a particle from the system. Homogeneous system:  $\mu = G/N (= g)$ i.e. Gibbs free energy per particle

### Link to statistical mechanics

A many-particle system will flow through its huge phase space, fluctuating through all microscopic states consistent with the constraints imposed on the system. For an isolated system with fixed energy E and fixed size V,N (microcanonic ensemble) these microscopic states are all equally likely at thermodynamic equilibrium (i.e. equilibrium is the most random situation).

- Partition function  $Z = Z(T, V) = \sum_{i} \exp(-E_i / k_B T) \implies$  Boltzmann-weighted sum over all possible system states

$$\Rightarrow$$
 F = -  $k_B T \ln(Z)$ 

- If groups of degrees of freedom are decoupled from each other (i.e. if the energetic states of one group do not depend on the state within the other group), then

$$Z_{\text{total}} = \left( \sum_{i} \exp(-E_{i}^{\text{A}} / k_{\text{B}} T) \right) \left( \sum_{i} \exp(-E_{i}^{\text{B}} / k_{\text{B}} T) \right) = Z^{\text{A}} Z^{\text{B}}$$

 $\Rightarrow$   $F_{\text{total}} = F^{A} + F^{B}$ 

e.g. electronic ↔ nuclear (Born-Oppenheimer) rotational ↔ vibrational

- N indistinguishable, independent particles:  $Z_{\text{total}} = 1/N! \left( Z_{\text{one particle}} \right)^N$ 

### **Computation of free energies: ideal gas I**

$$Z = 1/N! \left( X_{ucl} Z_{el} Z_{trans} Z_{rot} Z_{vib} \right)^{N}$$

$$\Rightarrow \mu(T,p) = G / N = (F + pV) / N = (-k_B T \ln(Z) + pV) / N$$

i) Electr. free energy  $Z_{el} = \sum_{i} \exp(-E_i^{el} / k_B T)$  Typical excitation energies  $eV >> k_B T$ ,  $\Rightarrow F_{el} \approx E^{tot} - k_B T \ln(I_{spin})$  Typical excitation energies  $eV >> k_B T$ , only (possibly degenerate) ground state contributes significantly

> Required input: Internal energy  $E^{\text{tot}}$ Ground state spin degeneracy  $I_{\text{spin}}$

ii) Transl. free energy  $Z_{\text{trans}} = \sum_{k} \exp(-\hbar k^2 / 2mk_B T)$  Particle in a box of length  $L = V^{1/3}$  $(L \rightarrow \infty) \implies Z_{\text{trans}} \approx V (2\pi mk_B T / \hbar^2)^{3/2}$ 

Required input: Particle mass *m* 

# Particle in box



 $\lambda_n = 2L/n$ 

$$E_n = n^2 \frac{h^2}{8mL^2}$$

1

$$\psi(z) \propto \sin\left(\frac{n\pi z}{L}\right).$$
## **Computation of free energies: ideal gas II**

 $\begin{array}{ll} \underline{\text{iii) Rotational free energy}} & Z_{\text{rot}} = \sum_{J} (2J+1) \exp(-J(J+1)B_{\text{o}} / k_{B}T) & \text{Rigid rotator} \\ \\ (\text{Diatomic molecule}) \Rightarrow & Z_{\text{rot}} \approx -k_{B}T & \ln(k_{B}T/\sigma B_{\text{o}}) & \sigma = 2 & (\text{homonucl.}), \\ & = 1 & (\text{heteronucl.}), \\ & B_{\text{o}} \sim md^{2} & (d = \text{bond length}) \end{array}$ 

Required input: Rotational constant  $B_0$ (exp: tabulated microwave data)

iv) Vibrational free energy  $Z_{vib}^{M} = \sum_{i=1}^{M} \sum_{n} \exp(-(n + \frac{1}{2})\hbar\omega_i / k_B T)$  Harmonic oscillator  $\Rightarrow \mu_{vib}(T) \stackrel{M}{=} \sum_{i=1}^{M} \frac{1}{2} \hbar\omega_i + k_B T \ln(1 - \exp(-\hbar\omega/k_B T))$ 

Required input:M fundamental vibr. modes  $\omega_i$ Calculate dynamic matrix $D_{ij} = (m_i m_j)^{-\frac{1}{2}} (\partial^2 E^{\text{tot}} \partial r_i \partial r_j)_{Leq}$ Solve eigenvalue problem $\det(D - 1 \ \omega_i^2)$ 

## **Computation of free energies: ideal gas III**



Alternatively:

 $\Delta \mu(T, p) = \Delta \mu(T, p^{\circ}) + kT \ln(p/p^{\circ})$ and  $\Delta \mu(T, p^{\circ} = 1 \text{ atm})$  tabulated in thermochem. tables (e.g. JANAF)

## **Computation of free energies: solids**

	$G(T,p) = E^{\text{tot}} + F^{\text{trans}} + F^{\text{rot}} + F^{\text{vib}} + F^{\text{conf}}$	+ <i>pV</i>
₽trans ₽ <sup>rot</sup>	Translational free energy Rotational free energy	$\propto 1/M \rightarrow 0$
рV	V = V(T,p) from equation of state, varies little	$\rightarrow$ 0 for <i>p</i> < 100 atm
Fconf	Configurational free energy	$\rightarrow$ Trouble maker
Etot	Internal energy	$\rightarrow DFT$
<b>F</b> <sup>vib</sup>	Vibrational free energy	→ phonon band structure
	<i>E</i> <sup>tot</sup> , <i>F</i> <sup>vib</sup> use differences use simple models to approx. <i>F</i> <sup>v</sup>	<sup>ib</sup> (Debye, Einstein)
	$\Rightarrow$ Solids (low T): $G(T,p) \sim E^{tot} + F^{conf}$	

II. Starting simple: Equilibrium concentration of point defects

> Solid State Physics, N.W. Ashcroft and N.D. Mermin, Holt-Saunders (1976)

## Isolated point defects and bulk dissolution

On entropic grounds there will always be a finite concentration of defects at finite temperature, even though the creation of a defect costs energy ( $E_D > 0$ ).

How large is it?

Internal energy: Config. entropy:  $E^{\text{tot}} = n E_{\text{D}}$   $F^{\text{conf}} = k_{\text{B}}T \ln Z(n)$ with  $Z = \frac{N (N-1) \dots (N-n-1)}{1 \cdot 2 \cdot \dots \cdot n} = \frac{N!}{(N-n)!n!}$ 

Minimize free energy:

 $(\partial G/\partial n)_{T,p} = \partial/\partial n_{T,p} (E^{\text{tot}} - F^{\text{conf}} + pV) = 0$ 

Forget *pV*, use Stirling: In  $N! \approx N(\ln N-1) \Rightarrow$ 

$$n/N = \exp(-E_{\rm D}/k_{\rm B}T)$$

N sites,

*n* defects (n << N)

III. Slightly more involved: Effect of a surrounding gas phase on the surface structure and composition

E. Kaxiras et al., Phys. Rev. B 35, 9625 (1987)

X.-G. Wang et al., Phys. Rev. Lett. 81, 1038 (1998)

## **Surface thermodynamics**

A surface can never be alone: there are always "two sides" to it !!! solid – gas solid – liquid solid – solid ("interface")

Phase I / phase II alone (bulk):

 $\begin{array}{l} G_{\mathrm{I}} = N_{\mathrm{I}} \; \mu_{\mathrm{I}} \\ G_{\mathrm{II}} = N_{\mathrm{II}} \; \mu_{\mathrm{II}} \end{array}$ 

Total system (with surface):

 $G_{\mathsf{I+II}} = G_\mathsf{I} + G_\mathsf{II} + \Delta G_\mathsf{surf}$ 

γA

$$\gamma = 1/A \left( G_{I+II} - \sum_{i} N_{i} \mu_{i} \right)$$

Surface tension (free energy per area)



## Example: Surface in contact with oxygen gas phase

$$\gamma_{\text{surf.}} = 1/A \left[ G_{\text{surf.}}(N_{\text{O}}, N_{\text{M}}) - N_{\text{O}} \mu_{\text{O}} - N_{\text{M}} \mu_{\text{M}} \right]$$



Use reservoirs: i)  $\mu_{O}$  from ideal gas ii)  $\mu_{M} = g_{M}^{bulk}$ 

Forget about  $F^{\text{vib}}$  and  $F^{\text{conf}}$  for the moment:  $\gamma(T,p) \approx (E^{(\text{slab})}_{\text{surf.}} - N_{\text{M}} E^{\text{ulk}}_{\text{M}})/A - N_{\text{O}} \mu_{\text{O}}(T,p) /A$ 

## **Surface Adsorption**

# Oxygen bond strength at basal TM surfaces



Strong variation with element and coverage





M. Todorova et al., Phys. Rev. Lett. 89, 096103 (2002)

## **Theory of adsorption : Newns-Anderson model**

 $\approx$  a generalization of molecular orbital (MO) schemes to finite bands



D.M. Newns, Phys. Rev. 178, 1123 (1969); P.W. Anderson, Phys. Rev. 124, 41 (1961)

### Theory of adsorption: Newns-Anderson model cont'd And Hammer-Norskov Model



Filling of *d*-band occupies antibonding O-M states



# Theory of adsorption: Newns-Anderson model cont'd





#### Achievements:

- qualitative electronic structure at the surface
- correct trend in adsorption strength over TM series
- reactivity patterns  $\rightarrow$  *d*-band center (Hammer/Nørskov)

#### Failures:

- no adsorbate induced reconstructions
- no site specificity ( $\rightarrow$  reactivity theory, HSAB, covalent/ionic)
- no lateral interactions

### Importance of intermediate bond strength

Low bond strength: no incentive to form/ bind intermediates High bond strength: no incentive to form products

### A good catalyst binds intermediates with medium bond strength

Sabatier principle (1913)





Two dimensional ultra-thin film a playground for manipulation of electron and chemistry

## Catalytically Active Gold: From Nano-particles to Ultrathin Films



Chen & Goodman Acc. Chem. Res. (2006)

#### **Origin:**

- 1. Low-Coordination Sites small may not be always good
- 2. Electronic Effects
- 3. Quantum size effects

"Unfortunately the data in the literature and the discussion vary widely; therefore the nature of the active Au species or structure or site remains unclear."

-- W. Goodman

Lack of precise preparation and characterization in general

# **Supported Catalyst**

### Structural Parameters



Kinetic Effects

## **Quantum Size Effects in Nanomaterials**



# **Electron Confinement in ultra thin film: Quantum Size Effect**



P.M. Echenique, J.B. Pendry, J. Phys. C: Solid State Phys. 11 (1978) 2056.

## **Quantum Well States from Quantum Mechanics**



### **Bound states in infinite or finite potential wells**

# 量子受限薄膜体系



 $\lambda_{F}$ 和 $v_{F}$ 与价电子数量和能带色散有关!

### 两个量子化电子态的能量差:

$\Delta = \pi \hbar  v_F$	/ Nd
---------------------------	------

薄膜厚度从N到N+1时的能谱移动:

 $\delta = 2d\Delta/\lambda_F$ 



metal/metal	metal/semicondutor
Ag/Cu(111)	Pb/Si(111)
Pb/Cu(111)	Pb/Ge(111)
Ag/Ni(111)	Pb/Ge(100)
Ag/Fe(100)	Ag/Si(111)
	AI/Si(111)
	Ag/GaAs(110)

# 通过改变薄膜的厚度来调控薄膜电子态及其催化性能

## Superconductivity Modulated by Quantum Size Effect: Pb/Si(111)







Q. K. Xue et. al Science 306, 1915(2004)

## **Tuning Chemical Reactivity by Quantum Well States**





# 金属表面化学反应活性的量子振荡

Ŷ 铅岛样品具有相同的结构、组成,唯一变化的参量是薄膜厚度!
 ↓ 氧气在的Pb薄膜的表面吸附和氧化反应随着薄膜厚度的变化出现量子振荡现象



X. C. Ma, W. X. Li, Q. K. Xue et al, Proceeding of National Academy of Sciences (2007)

# 金属表面化学反应活性的量子振荡



# Effects of quantum well states: Fe adatom on Pb/Si



n (Fe islands)<sub>11,13,15</sub> > n (Fe islands)<sub>10,12,14,16</sub>

#### Density oscillation of Fe islands on QWS Pb films

L.Y. Ma, J.F. Jia, Q.K. Xue, et al., Phys. Rev. Lett. 97 (2006) 266102.



# 隧道扫描电镜(STM) 测量





dI/dV





Symmetry broken at surface affects Pz orbital Gamma is dominated by Pz orbial

## Pb(111) 膜的UPS、及其随厚度的调制



2

15

角分辨UPS谱 (Γ:发向UPS)





只有在发向UPS(布里渊区中心处)表现 出量子尺寸调控效应 (Pz orbital)



# 表面电子密度的空间延展性/衰减长度





# QSE对化学反应活性调制的起源

- 振荡的原因:费米能级处态密度以及电子波函数在真空衰减的快慢随膜的厚度调制而变化
- 二维薄膜原子轨道的对称性破缺决定了薄膜的量子尺寸效应
  及其表面活性

0.32

0.34

0.36

0.38

0.40

0.42

0.44

0.46

0.48

电子在动量空间的三维衰减图





在动量空间Gamma点处电子衰减最慢, 其相应的轨道对称性为P<sub>z</sub>轨道。

## 量子尺寸效应: <sup>局域功函数、HOQWS & LUQWS (STM)</sup>



4

15



$$\phi_{LWF} = 0.952 [d \ln I/dz]^2$$
$$\approx 0.952 [d \ln \rho/dz]^2$$



# XPS + DFT

# Oxidation of Pt(332) and CO Oxidation

# **Effect of Step Edge**



# **NO Dissociation at Ru Steps**



T.Zambelli, J. Wintterlin, J. Trost, G. Ertl, Science **273**, 1688 (1996).





- High  $\epsilon_d$ 's at steps.
- More active metal atoms.

Hammer (1999).

# X-ray Photoelectron Spectroscopy (XPS)



$$hv = E_{kin} + \phi + \mathcal{E}_c$$
$$\phi = E_V - E_F$$

Core-electron Energy

$$\varepsilon_c = E(n-1) - E(n)$$

E(n) / E(n-1): total energy before / after core excited (frozen core approximation, valence electron relaxation included)

#### Surface Core Level Shift

$$\Delta_{SCLS} = \varepsilon_c^{surf} - \varepsilon_c^{bulk}$$

Final state effect included

**Element specific and geometric sensitive Fingerprint as adsorption site and oxidation state**
### Clean surfaces Rh(553)



Rh 3d 5/2 from DFT

	SCLS (meV)
S	0
U	530
T-1	310
<b>T-2</b>	350
<b>T-3</b>	370



## **CO** Adsorption on Pt(111)

Top Bridge





## **CO Adsorption on Pt(332)**



J.G. Wang, W. X. Li et al, Phys. Rev. Lett. 95, 256102 (2005)

## O<sub>2</sub> Adsorption on Pt(332) and Pt(111)

Oxygen 1s binding energy



O1s binding energy: 529.75 eV for Pt(111) 529.55 eV for Pt(332) Difference: 0.20 eV

Pt(111) saturated at 0.25 ML Pt(332) saturated at 0.42 ML

## **1D PtO<sub>2</sub> along Step Edge of Pt(332)**





## **CO Oxidation on O pre-covered Pt(332)**



CO



reaction negligible at 220 K on Pt(332), instead of 270 K on Pt(111)



J.G. Wang, W. X. Li et al, Phys. Rev. Lett. 95, 256102 (2005)

## **CO Oxidation on Pt(332)**

#### **Initial State**







Reaction Barrier for CO oxidation:0.60 eV for 1D PtO2 $(E_b=0.77 \text{ eV}$  for step oxygen)0.71 eV for terrace oxygen ( $E_b=0.54 \text{ eV}$  for terrace oxygen)

#### DFT + Fast STM

### Two step reaction on a nanostructured surface: (10×2)-0/Rh(110)

high local oxygen coverage

strained

nanostructured

C. Africh, F. Esch, W. X. Li, M. Corso, B. Hammer, R. Rosei, and G. Comelli1, *Phys. Rev. Lett.* 93, 126104 (2004)

#### Formation of the $(10\times2)$ structure

29L O<sub>2</sub> (200K)

annealed

to 530 K

(1×1)

+0.5ML O

T>100° C



#### (2×2)p2mg





 $8 \times 9 \text{ nm}^2$ 

- (1×2) *missing row* - O *threefold* in *zig-zag*  (10×2)



40×30 nm<sup>2</sup>

#### Characteristics of the $(10\times2)$ structure



(2×1)p2mg

Height

## **Two-Step Reaction**

Room temperature, partial pressure of H<sub>2</sub>: 10<sup>-8</sup> mbar



40x30nm<sup>2</sup> Acquisition time: 35 s per image

## 1st-step reaction



## 2nd-step reaction

Brighter areas where zig-zag oxygen reacted off

 $150 \times 150 \text{ Å}^2$ 

Final surface: clean, segmented (1x2)



2<sup>nd</sup> oxygen removal:

- homogeneous nucleation

(2×2)p2mg segmented



1<sup>st</sup> oxygen removal:

- reaction front
- zig-zag oxygen left
- strain removal

## H<sub>2</sub> Dissociation on Rh(110)-(1x2) Barrier: 0.09 eV







## **1st Step eaction**

H<sub>2</sub> dissociation barrier:
island edge: 0.23 eV
middle part and RV between segments: larger than 0.67 eV



## **1st Step eaction**

H<sub>2</sub> dissociation barrier:
island edge: 0.23 eV
middle part and RV between segments: larger than 0.67 eV

#### OH formation barrier:

- island edge: 0.78 eV
- middle part and RV between segments: 0.92 eV and 1.16 eV



## Wavefront Formation



O's Stability: •Site A: 1.85 eV •Site B: 1.85 eV •Site C: 1.36 eV •Site D: 1.98 eV

zig-zag structure preferred









## Wavefront Formation

## 2nd Step Reaction



2<sup>nd</sup> oxygen removal: - homogeneous nucleation H2 dissociates barrier: 0.12 eV H atom can only adsorb between the ridge

### Bridge "pressure" and "materials" gap

### First-principles, atomistic thermodynamics

### Pressure / materials gap



#### Oxidation catalysis:

possible oxide formation at the catalyst surface

Low vs. high temperatures

Finite dosage vs. gas phase reservoir



H. Over et al., Science 287, 1474 (2000)

## Ordered Structures: O/Ag(111)

#### (4x4)-0 (Carlisle et al, PRL, 2000)





#### Schmid et al, Phy. Rev. Lett. 96, 146102 (2006)



Schnadt et al, Phy. Rev. Lett. 96, 146101 (2006)



## From UHV to Real World (Theoretical Recipes) Ab initio atomistic thermodynamics

Gibbs Formation Energy of Adsorption

 $G = 1/A \left[ G_{surf.}(N_{O}, N_{M}) - N_{O} \mu_{O} - N_{M} \mu_{M} \right]$   $O_{2} gas$  Use reservoirs:  $i) \mu_{O} \text{ from ideal gas}$   $ii) \mu_{M} = g_{M}^{bulk}$ 

Forget about  $F^{\text{vib}}$  and  $F^{\text{conf}}$  for the moment:  $\gamma(T,p) \approx (E^{(\text{slab})}_{\text{surf.}} - N_{\text{M}} E^{\text{ulk}}_{\text{M}})/A - N_{\text{O}} \mu_{\text{O}}(T,p) /A$ 

## Average Gibbs free energy of adsorption



W. X. Li et al. Phys. Rev. Lett. 90, 256102 (2003)

# Pressure – Temperature Phase Diagram: O/Ag(111) bases on ab initio atomistic thermodynamics



W. X. Li et al. Phys. Rev. Lett. 90, 256102 (2003)

## Can DFT do more than we expect ?

## Ammonia Synthesis from First Principles Calculations

Honkala et al. Science **307**,555 (2005)

## Catalytic ammonia synthesis $N_2+3H_2 \rightarrow 2NH_3$



Ozaki and Aika, Catalysis 1 (Anderson and Boudart, Ed.)

## **Reaction mechanism**

#### Reaction mechanism

At equilibrium

$$N_{2} + 2^{*} \rightarrow 2N^{*}$$

$$N^{*} + H^{*} \leftrightarrow NH^{*} + *$$

$$NH^{*} + H \leftrightarrow NH_{2}^{*} + *$$

$$NH_{2}^{*} + H^{*} \leftrightarrow NH_{3}^{*} + *$$

$$NH_{3}^{*} \leftrightarrow NH_{3} + *$$

$$H_{2} + 2^{*} \leftrightarrow 2H^{*}$$

Rate-limiting step

## Ammonia synthesis over Ru

Logadottir, Nørskov, J. Catal. 220, 273-279 (2003)



## Steps do everything

Au decorates steps: Hwang, Schroder, Gunther, Behm, Phys. Rev. Lett. 67, 3279 (1991)





Dahl, Logadottir, Egeberg, Larsen, Chorkendorff, Törnqvist, Nørskov, Phys.Rev.Lett. 83, 1814 (1999)

## Rates of elementary steps

#### Harmonic transition state theory:



#### Rate-limiting step

Emmett, Brunauer, JACS **55** 1738 (1933)



WIGNOV.

# Adsorption geometries for different intermediates

#### $\Delta E$ =-34.3 kJ/mol



 $\Delta E$ =-37.6 kJ/mol



 $\Delta E=-97.0 \text{kJ/mol}$ 



 $\Delta E=-167.8 \text{ kJ/mol}$ 

∆E=-178.2 kJ/mol





## Different N<sub>2</sub> transition state geometries











D









## **Kinetic model**

Reaction rate per site:

$$r_{tot}(T, p_{N_2}, p_{H_2}, p_{NH_3}) = (1 - \gamma)r^f$$

$$r^f = \sum_i P_i k_i p_{N_2}$$

$$\gamma = \frac{p_{NH_3}^2}{K_g p_{N_2} p_{H_2}^3}$$

Harmonic transition state theory:

$$k_i = \nu \exp(-E_a / k_B T)$$

P<sub>i</sub> is the probability to have a certain local environment

#### Total rate through the reactor:

$$\boldsymbol{\gamma}_{PFR}^{total} = \sum_{j=0}^{N} r(T, p_{N_2}^j, p_{H_2}^j, p_{NH_3}^j) \times \frac{\rho \mathbf{m}}{\mathbf{N}}$$
### How to get the number of active sites

#### **A real ammonia synthesis catalysts**: TEM figure of a Ru particle over spinel support





A. Carlsson, S. Dahl, Haldor Topsøe A/S

# From size distibution to number of active sites: $\rho$

#### Wullf construction based on DFT:



A typical calculated Ru particle with an average diameter 2.9 nm.

Honkala et al. Science 307,555 (2005)

I. Remediakis

# Ab initio prediction of the rate of the real catalyst

Experimental data: S Dahl & C. Christensen, Haldor Topsøe AS



Total pressure 100 bar, H<sub>2</sub>:N<sub>2</sub>=3:1

Honkala et al. Science **307**,555 (2005)

### Summary

- 1. ab initio prediction for NH<sub>3</sub> productivity on the supported metal particle
- 2. The absolute error in DFT calculations is about 0.2-0.3 eV but the relative errors are in in order of 0.06 eV ( $\sim k_BT$  at reaction conditions).
- 3. The build-in insensitivity to absolute errors together with the higher accuracy of the DFT methods for relative energies, offers hope that DFT calculations can give a good overall description of the catalytic activity of other reactions as well

### Can DFT do more than we expect ?

### Selectivity

Greely &Norskov, Nature Materials (2006)

#### **Promotional Effect of Gold in Catalysis by Palladium-Gold**



Goodman believes that selectivity is increased because the unwanted reactions need more than two Pd atoms.

### $CH_3COOH + CH_2CH_2 + 1/2 O_2 \rightarrow CH_3COOCH = CH_2 + H_2O$

on Pd/Au alloy better selectivity for vinyl acetate than on Pd (Wayne Goodman, Science (2005))



### Methane Dehydrogenation on Rh@Cu(111)



First C-H bond breaking is rate limiting step
CH3 → CH2 + H: require bridge site

A. Kokalj et al JACS (2006)

### Methane Dehydrogenation on Rh@Cu(111)



Second dehydrogenation of methane is prevented
C-C coupling or formation of methanol may be preferential

A. Kokalj et al JACS (2006)

# Controlling the catalytic bond-breaking selectivity of Ni surfaces by step blocking

C <sub>2</sub> H <sub>4</sub> Activation and Selec	ctivity	
C - H bond breaking :		Dissociated ethylene
$C_2H_4 = C_2H_3 + H_3$	H	and the second
$C_2H_3 = C_2H_2 + H_3$	H	20nmv20nm
C - C coupling (	Ni(211) and Ni(111))	- 2011112201111
C - C bond breaking		
$C_{2}H_{4} = 2CH_{2}$		Ag atoms /
$CH_2 = C + 2H$		
C coke (I	Ni(211))	

40nmx40nm

R. T. Vang, J. K. Norskov and F. Besenbacher et al, Nature Materials (2005)

Blocking Ni step with Ag prevents carbon formation

# Controlling the catalytic bond-breaking selectivity of Ni surfaces by step blocking



R. T. Vang, J. K. Norskov and F. Besenbacher et al, Nature Materials (2005)

P-F

C-H bond breaking



C-C bond breaking

### Can DFT do more than we expect ?

Computational high-throughput screening of electrocatalytic materials for hydrogen evolution

Greely &Norskov, Nature Materials (2006)

## Hydrogen driven Fuel Cell



 $2H_2 \rightarrow 4H^+ + 4e^-$ Cathode:

 $O_2 + 4H^+ + 4e^- \rightarrow 2H_2O$ 

 $2H_2+O_2 \rightarrow 2H_2O$  + Heat

Electricity

Heat





#### **Environment Effects**

**BiP** 

Surface Segregation Island Formation Water splitting and oxygen Metal dissolution





### Thank you for your attention

